

# Quantum phase transitions in $d$ - wave superconductors

Ki-Seok Kim, Sung-Sik Lee, Jae-Hyeon Eom and Sung-Ho Suck Salk<sup>1</sup>

*Department of Physics, Pohang University of Science and Technology Pohang 790-784, Korea*

<sup>1</sup>*Korea Institute of Advanced Studies, Seoul 130-012, Korea*

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Based on an effective Lagrangian obtained from the holon-pair boson theory of Lee and Salk [Phys. Rev. B **64**, 052501 (2001)] for high  $T_c$  cuprates, we explore physical states involved with quantum phase transitions around a critical hole doping of  $d$  - wave superconductivity. We find a new quantum phase transition, a confinement to deconfinement transition for internal gauge charge in the superconducting phase. An antiferro- to para- magnetic transition in the superconducting state is explained in the context of the confinement to deconfinement transition. We obtain effective field theories in each region associated with the phase transitions.

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Since experimental revelation of weak antiferromagnetism in underdoped superconducting state[1], the coexistence of the antiferromagnetism ( $AFM$ ) and  $d$  - wave superconductivity ( $dSC$ ) has been a subject of great theoretical interest[2]. However, it is still unknown whether despite the coexistence there exists any interplay (or correlation) between the  $AFM$  and  $dSC$ . Earlier we found that the  $AFM$  of nodal fermions can coexist with the  $dSC$  with no interplay in very low doping region[3]. As a result the superconducting transition in the underdoped region is found to fall in the XY universality class in agreement with experiments[3, 4, 5]. It is naturally expected that the  $AFM$  disappears as hole concentration increases. It is well known that in the overdoped region there exists no  $AFM$ . Thus a quantum phase transition associated with the  $AFM$  is expected to occur in the superconducting phase. Extending the earlier analysis[3] to higher doping where the  $AFM$  is expected to disappear, we show that a confinement to deconfinement transition ( $CDT$ ) of internal  $U(1)$  gauge charge can occur in the superconducting phase. This new quantum phase transition is shown to be responsible for the disappearance of the  $AFM$  in the overdoped region[1]. Further, we argue that non-Fermi liquid behavior in high  $T_c$  cuprates can be resolved in the quantum critical point associated with the  $CDT$ .

Earlier, the holon-pair boson theory of Lee and Salk[6] reproduced the salient features of the observed arch-shaped superconducting transition temperature in the phase diagram of hole doped high  $T_c$  cuprates and the peak-dip-hump structure of optical conductivity in agreement with observations. Following this holon-pair boson theory[6], we introduce an effective Lagrangian[3] involved with low energy excitations. Here the low energy excitations refer to the phase fluctuations of the spinon pair and holon pair order parameters, the massless spinons (Dirac fermions near the  $d$  - wave nodal points) and holons (bosons), and gauge fluctuations associated with these particles. The gauge fluctuations  $a_\mu$  allow the presence of internal flux responsible for energy lowering, which arises as a result of electron hopping. Considering the above elementary excitations, we write the  $(2 + 1)D$

low energy Lagrangian in the slave boson representation,

$$\begin{aligned} Z &= \int D\psi_l D\psi_b D\phi_{sp} D\phi_{bp} Da_\mu e^{-\int d^3x \mathcal{L}}, \\ \mathcal{L} &= \mathcal{L}_{sp} + \mathcal{L}_{bp}, \\ \mathcal{L}_{sp} &= \frac{K_{sp}}{2} |\partial_\mu \phi_{sp} - 2a_\mu|^2 + \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu) \psi_l, \\ \mathcal{L}_{bp} &= \frac{K_{bp}}{2} |\partial_\mu \phi_{bp} - 2a_\mu - 2A_\mu|^2 + |(\partial_\mu - ia_\mu - iA_\mu) \psi_b|^2 \end{aligned}$$

Here  $\phi_{sp}$  ( $\phi_{bp}$ ) is the phase field of the spinon pair (holon pair) order parameter.  $K_{sp} \sim J_\delta |\Delta_{sp}|^2$  is the phase stiffness of the spinon pair order parameter and  $K_{bp} \sim J |\Delta_{sp}|^2 |\Delta_{bp}|^2$ , that of the holon pair order parameter where  $J$  is the antiferromagnetic coupling and  $J_\delta = J(1 - \delta)^2$ , the renormalized antiferromagnetic coupling in association with hole doping  $\delta$ [6].  $\Delta_{sp(bp)} = |\Delta_{sp(bp)}| e^{i\phi_{sp(bp)}}$  is the spinon (holon) pairing order parameter.  $\psi_l$  represents the 4 component spinor[7, 8] of the massless Dirac fermion near the nodal points ( $l = 1$  and 2) and  $\psi_b$ , the holon (boson) quasiparticle.  $a_\mu$  is the  $U(1)$  internal gauge field and  $A_\mu$ , the external electromagnetic field[4, 5].

Defining  $\phi_p = \phi_{bp} - \phi_{sp}$  and  $\phi_c = -(\phi_{bp} + \phi_{sp})$ [3], Eq. (1) is rewritten,

$$\begin{aligned} Z &= \int D\psi_l D\psi_b D\phi_p D\phi_c Da_\mu e^{-\int d^3x \mathcal{L}}, \\ \mathcal{L} &= \mathcal{L}_c + \mathcal{L}_p + \mathcal{L}_{int}, \\ \mathcal{L}_c &= \frac{\kappa}{2} |\partial_\mu \phi_c + 4a_\mu + 2A_\mu|^2 + \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu) \psi_l \\ &\quad + |(\partial_\mu - ia_\mu - iA_\mu) \psi_b|^2, \\ \mathcal{L}_p &= \frac{\kappa}{2} |\partial_\mu \phi_p - 2A_\mu|^2, \\ \mathcal{L}_{int} &= \kappa_{cp} (\partial_\mu \phi_c + 4a_\mu + 2A_\mu) (\partial_\mu \phi_p - 2A_\mu), \quad (2) \end{aligned}$$

where  $\kappa = \frac{K_{sp} + K_{bp}}{4}$  and  $\kappa_{cp} = \frac{K_{sp} - K_{bp}}{4}$ . The original Lagrangian Eq. (1) is now rewritten in terms of the Cooper pair phase field,  $\phi_p = \phi_{bp} - \phi_{sp}$  as a composite of the spinon pair phase field  $\phi_{sp}$  and the holon pair phase field  $\phi_{bp}$ , and the "chargeon pair" phase field,  $\phi_c = -(\phi_{bp} + \phi_{sp})$  as a composite of the spinon pair phase

field  $\phi_{sp}$  and the "anti-holon" pair phase field  $-\phi_{bp}$ [3]. Here "chargeon pair" refers to the spin singlet pair oppositely charged to the Cooper pair and "anti-holon", the opposite charge of holon with spin 0. The Cooper pair as a hole pair in the hole doped cuprates carries charge  $+2e$  and no internal gauge charge while the chargeon pair carries charge  $-2e$  and internal gauge charge  $-4\tilde{e}$ . Here  $\kappa$  is the phase stiffness of both the chargeon pair order parameter  $\phi_c$  and the Cooper pair order parameter  $\phi_p$ .

Making a renormalization group analysis, it can be shown that via the coupling term  $\mathcal{L}_{int}$  the phase stiffness  $\kappa$  is renormalized in low energy limit[3, 9], and the low energy effective Lagrangian Eq. (2) can be rewritten as

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_c + \mathcal{L}_p, \\ \mathcal{L}_c &= \frac{K}{2} |\partial_\mu \phi_c + 4a_\mu + 2A_\mu|^2 + \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu) \psi_l \\ &\quad + |(\partial_\mu - ia_\mu - iA_\mu) \psi_b|^2, \\ \mathcal{L}_p &= \frac{K}{2} |\partial_\mu \phi_p - 2A_\mu|^2,\end{aligned}\quad (3)$$

with  $K = \frac{K_{sp}K_{bp}}{K_{sp}+K_{bp}}$  where  $K$  is the renormalized phase stiffness via  $\mathcal{L}_{int}$ . The effective Lagrangian above is now separated into two independent sectors, one for the chargeon pair Lagrangian and the other for the Cooper pair Lagrangian. The former explains for the internal gauge charge  $CDT$ [10, 11, 12, 13] (as will be discussed later) and the latter, for the superconducting phase transition. We note that both the Dirac fermion and the holon (boson) are coupled to the chargeon pair field via the  $U(1)$  internal gauge field.

As hole concentration increases in the underdoped region, the phase stiffness increases in this region[3]. When the hole concentration exceeds a critical value  $\delta_c$  of  $d$ -wave superconducting transition, the Cooper pairs are bose-condensed, i.e.,  $\langle e^{i\phi_p} \rangle \neq 0$  and the superconducting state emerges. The insulator-superconductor transition at  $T = 0K$  falls into the quantum phase transition of the XY universality class in the extreme type II limit[3]. At finite temperature the KT transition is expected to occur[4]. See Fig. 1 for a schematic diagram of showing the regions of the Cooper pair bose condensation  $\langle e^{i\phi_p} \rangle \neq 0$  and the chargeon pair condensation  $\langle e^{i\phi_c} \rangle \neq 0$  as a function of hole doping  $\delta$ .

Integrating over the Dirac spinon field and the holon field in  $\mathcal{L}_c$ , we obtain in the absence of electromagnetic field  $A_\mu$ ,

$$\mathcal{L}_c = \frac{K}{2} |\partial_\mu \phi_c + 4a_\mu|^2 + \frac{1}{2g_{eff}} (\partial \times a) \frac{1}{\sqrt{-\partial^2}} (\partial \times a)$$

where  $g_s = \frac{8}{N_s}$ [14],  $g_b = \frac{8}{N_b}$ [15] and  $g_{eff} = \frac{g_b g_s}{g_b + g_s}$ . Here  $N_s$  is the flavor number of the Dirac fermions and  $N_b$ , the flavor number of the holon quasiparticles. At the critical hole concentration  $\delta_c$  where the superconducting phase transition occurs, the internal gauge charge  $CDT$  may not necessarily occur, as schematically shown in Fig.

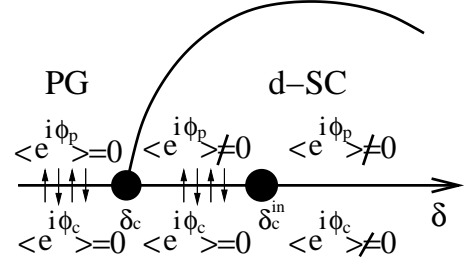


FIG. 1: Schematic phase diagram of underdoped high  $T_c$  cuprates at low energy in the plane of hole doping  $\delta$  vs. temperature  $T$ . Only the pseudogap phase (PG) and  $d$ -wave superconducting phase (d-SC) are partially shown.

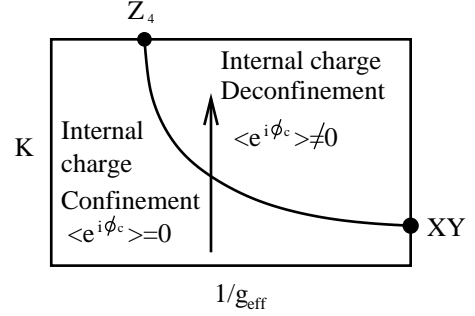


FIG. 2: Schematic phase diagram of the  $(2+1)D$  Abelian Higgs model with multiple charge  $4\tilde{e}$  in the plane of the phase stiffness  $K$  of the chargeon pair vs. the effective coupling strength  $g_{eff}$  shown in Eq. (4).

2[10, 11, 12, 13]. It may be possible to have the internal charge  $CDT$  at a critical value  $\delta_c^{in}$  of hole doping above  $\delta_c$  where a critical value  $K_c$  of the chargeon pair phase stiffness is reached to cause the condensation of chargeon pair bosons, i.e.,  $\langle e^{i\phi_c} \rangle \neq 0$  and the suppression of the internal gauge fluctuations  $a_\mu$ . On the other hand, below the critical phase stiffness  $K_c$  (the phase boundary line in Fig. 2) the internal charge confinement phase will result in[16].

In passing, we briefly discuss the coexistence between the  $AFM$  and  $dSC$ [3] for completeness. In the underdoped region of  $\delta_c < \delta < \delta_c^{in}$  the  $dSC$  coexists with the confinement phase of the internal charge  $\tilde{e}$  as is schematically displayed in Fig. 2. The chargeon pair becomes disordered, i.e.,  $\langle e^{i\phi_c} \rangle = 0$  below the critical phase stiffness  $K_c$  in this low hole doping region. This leads to  $AFM$  of the nodal fermions, as will be discussed here. Integrating over  $\phi_c$  in the disordered phase of  $\langle e^{i\phi_c} \rangle = 0$ , we obtain from Eq. (3),

$$\begin{aligned}\mathcal{L}_c &= \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu) \psi_l + |(\partial_\mu - ia_\mu - iA_\mu) \psi_b|^2 \\ &\quad + \frac{1}{2g} |4\partial \times a + 2\partial \times A|^2\end{aligned}\quad (5)$$

with the coupling strength  $g \sim K_c - K$ [3]. Neglecting coupling between the holon field and the gauge field  $a_\mu$  for the time being, the Dirac fermion field is known to be massive for  $N_s < N_c \approx 3.24$  in the case of non-compact

U(1) gauge field[3, 7, 8, 14, 17] where  $N_s$  is the flavor number of the Dirac fermion and  $N_c$ , the critical flavor number for chiral symmetry breaking. The massive Dirac fermion leads to *AFM*[3, 7, 14]. The magnetization  $m$  is proportional to the effective coupling strength  $g$ [7, 14]. Thus it is given by  $m \sim (K_c - K) \sim (\delta_c^{in2} - \delta^2)$ [3]. The chargeon pair sector and the Cooper pair sector are decoupled as shown in the effective Lagrangian Eq. (3). This implies that the *AFM* of the nodal fermions coexists with the *dSC* by having no direct coupling (interplay) between the two[3]. Now taking into account the contribution of the holon field coupled to the internal gauge field, the critical flavor number  $N_c \approx 3.24$  of the chiral symmetry breaking is reduced to  $N'_c \approx 2.24$ [18]. The flavor number  $N_s = 2$  of present interest is close to the critical flavor number. Admitting the compactness of the internal U(1) gauge field, confinement of the internal U(1) gauge charge is not clearly understood yet. Recently, Hermele et. al claimed that critical fluctuations of massless particles can result in deconfinement of the internal charge in the limit of large flavor numbers[19]. In our case the flavor number of quasiparticles is small and thus the deconfinement is not expected to occur from the massless quasiparticle excitations. Instead particle-hole bound states are expected to occur, causing a dynamically generated mass to both the spinons and holons. To determine the critical flavor number in the presence of instantons remains as an open issue.

Using the unitary gauge  $4\tilde{a}_\mu = \partial_\mu \phi_c + 4a_\mu + 2A_\mu$  in Eq. (3), we obtain for  $\mathcal{L}_c$

$$\begin{aligned} \mathcal{L}_c = & \bar{\psi}_l \gamma_\mu (\partial_\mu + i\frac{1}{4}\partial_\mu \phi_c - i\tilde{a}_\mu + i\frac{1}{2}A_\mu) \psi_l \\ & + |(\partial_\mu + i\frac{1}{4}\partial_\mu \phi_c + i\tilde{a}_\mu - i\frac{1}{2}A_\mu) \psi_b|^2 + \frac{K}{2} |4\tilde{a}_\mu|^2. \end{aligned} \quad (6)$$

In the large limit of stiffness parameter,  $K \rightarrow \infty$ , the gauge fluctuations have  $Z_4$  symmetry[10, 11, 12, 13]. Owing to the internal gauge charge  $4\tilde{e}$  (see the last term in Eq. (6)) the effective field theory of the quasiparticles is involved with the  $Z_4$  gauge theory in association with the Dirac spinon and the holon. In the deconfinement phase the  $Z_4$  gauge fluctuations are suppressed and coupling between the Dirac spinon and the holon via the  $Z_4$  gauge field becomes negligible[11]. Thus both the massless Dirac fermion and the massless holon become free. The chiral symmetry breaking is not expected to occur. Introducing the renormalized spinon (quasi-spinon) field  $\psi_{cf} = e^{i\frac{1}{4}\phi_c} \psi_l$  and the renormalized holon (quasi-holon) field  $\psi_{cb} = e^{i\frac{1}{4}\phi_c} \psi_b$  in the above Lagrangian, an effective Lagrangian in the deconfinement phase is obtained to be

$$\mathcal{L}_c = \bar{\psi}_{cf} \gamma_\mu (\partial_\mu + i\frac{1}{2}A_\mu) \psi_{cf} + |(\partial_\mu - i\frac{1}{2}A_\mu) \psi_{cb}|^2. \quad (7)$$

As can be seen from this Lagrangian, the quasi-spinon carries the fractional charge  $-e/2$  and the quasi-holon, the fractional charge  $+e/2$ . Thus it is possible that in the deconfinement phase charge fractionalization may occur.

This deconfinement phase corresponds to a fractionalized metallic state of massless quasiparticles. The observed antiferromagnetic to paramagnetic transition[1] can be understood in the context of the *CDT*. Various physical states involved with quantum phase transitions are summarized in both Table 1 and Fig. 1.

At the critical point  $\delta_c^{in}$  Algebraic Fermi liquid is expected to occur since instanton excitations become irrelevant owing to the gauge charge  $4\tilde{e}$  of the chargeon pair field[20]. Integrating over the chargeon pair field at the critical point in Eq. (3) and incorporating the gauge shift of  $a_\mu \rightarrow a_\mu - \frac{1}{2}A_\mu$ , we obtain *QED*<sub>3</sub> in terms of the massless Dirac spinon and massless holon coupled to the non-compact internal U(1) gauge field,

$$\begin{aligned} \mathcal{L}_c = & \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu + i\frac{1}{2}A_\mu) \psi_l + |(\partial_\mu - ia_\mu - i\frac{1}{2}A_\mu) \psi_b|^2 \\ & + \frac{N_a}{16} (\partial \times a) \frac{1}{\sqrt{-\partial^2}} (\partial \times a), \end{aligned} \quad (8)$$

where  $N_a$  is the flavor number of the chargeon pair field. Here  $N_a = 1$ . Even if Eq. (8) is similar to Eq. (5), the internal U(1) gauge field is non-compact in Eq. (8) while it is compact in Eq. (5). The chiral symmetry breaking is not expected to occur since the coupling strength is above the critical value, i.e.,  $N_a + N_s + N_b > N_c$ . This result is consistent with the fact that the mass gap of the Dirac fermion or the magnetization vanishes at the critical hole doping  $\delta_c^{in}$  ( $m \sim \delta_c^{in2} - \delta^2$ ). As well known, there exist no well defined quasiparticles in this *QED*<sub>3</sub> owing to massless gauge fluctuations[14]. But in the deconfinement phase the renormalized spinon and holon are expected to be well defined quasiparticles because the  $Z_4$  gauge fluctuations are massive and suppressed. This seems to be the main difference between critical metallic state at the critical point and fractionalized metallic state in the deconfinement phase. Our non-compact *QED*<sub>3</sub> emerging at the critical point in the superconducting state may explain non-Fermi liquid behavior[21] in a completely different context from the previous studies[22, 23, 24]. This non-Fermi liquid behavior is not related with the superconductivity but associated with the *CDT*.

We showed that there exists no interplay between the antiferromagnetism and superconductivity despite the coexistence. This results in the fact that the superconducting transition falls into the XY universality class in agreement with experiments[5]. In addition, we found that the antiferro- to para- magnetic transition in the superconducting state[1] can be resolved by the confinement to deconfinement transition of the internal gauge charge. Further, we argued that non-Fermi liquid behaviors in high  $T_c$  cuprates may be described by a critical field theory (*QED*<sub>3</sub>) in association with the new quantum critical point.

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TABLE I: Physical states in association with quantum phase transitions

	Pseudogap ( $\delta < \delta_c$ )	Superconductivity ( $\delta_c < \delta < \delta_c^{in}$ )	Superconductivity ( $\delta_c^{in} < \delta$ )
Phase	$\langle e^{i\phi_p} \rangle = 0$ $\langle e^{i\phi_c} \rangle = 0$ $\langle \bar{\psi}_l \psi_l \rangle \neq 0$	$\langle e^{i\phi_p} \rangle \neq 0$ $\langle e^{i\phi_c} \rangle = 0$ $\langle \bar{\psi}_l \psi_l \rangle \neq 0$	$\langle e^{i\phi_p} \rangle \neq 0$ $\langle e^{i\phi_c} \rangle \neq 0$ $\langle \bar{\psi}_l \psi_l \rangle = 0$
Physical state	Mott insulator Confinement Antiferromagnetism	Superconductor Confinement Antiferromagnetism	Superconductor Deconfinement Paramagnetism

- [1] J. E. Sonier et al., Science **292**, 1692 (2001); Y. Sidis et al., Phys. Rev. Lett. **86**, 4100 (2001); H. A. Mook et al., Phys. Rev. B **64**, 012502 (2001); M. -H. Julien et al., Phys. Rev. B **63**, 144508 (2001); S. Ono et al., Phys. Rev. Lett. **85**, 638 (2000); Ch. Niedermayer et al., Phys. Rev. Lett. **80**, 3843 (1998).
- [2] S. C. Zhang et al. Phys. Rev. B **60**, 13070 (1999); C. N. A. van Duin and J. Zaanen, Phys. Rev. B **61**, 3676 (2000); M. Granath et al., Phys. Rev. Lett. **87**, 167011 (2001); S. Sachdev, Rev. Mod. Phys. **75**, 913 (2003); Kwon Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001); T. Pereg-Barnea and M. Franz, Phys. Rev. B **67**, 060503(R) (2003); E. Demler et al., Phys. Rev. B **65**, 155103 (2002); Bumsoo Kyung, Phys. Rev. B **62**, 9083 (2000); Wei-Min Zhang, Phys. Rev. B **65**, 104513 (2002); T. Takimoto and T. Moriya, Phys. Rev. B **66**, 134516 (2002); S. Sorella et al., Phys. Rev. Lett. **88**, 117002 (2002); A. I. Lichtenstein and M. I. Katsnelson, Phys. Rev. B **62**, R9283 (2000).
- [3] Ki-Seok Kim, Sung-Sik Lee, Jae-Hyeon Eom and Sung-Ho Suck Salk, cond-mat/0312667; Phys. Rev. B submitted.
- [4] Ki-Seok Kim, Jae-Hyeon Eom, Young-Il Seo, and Sung-Ho Suck Salk, Phys. Rev. B **69**, 014504 (2004).
- [5] F. S. Nogueira, Phys. Rev. B **62**, 14559 (2000); references therein; Dominic J. Lee and I. D. Lawrie, Phys. Rev. B **64**, 184506 (2001); references therein.
- [6] S.-S. Lee and S.-H. S. Salk, Phys. Rev. B **64**, 052501 (2001); S.-S. Lee and S.-H. S. Salk, Phys. Rev. B **66**, 054427 (2002); S.-S. Lee, J.-H. Eom, K.-S. Kim, and S.-H. Suck Salk, Phys. Rev. B **66**, 064520 (2002).
- [7] I. F. Herbut, Phys. Rev. Lett. **88**, 047006 (2002); I. F. Herbut, Phys. Rev. B **66**, 094504 (2002); references therein.
- [8] Z. Tesanovic et al., Phys. Rev. B **65**, 180511 (2002).
- [9] Performing a standard duality transformation of Eq. (2), we obtain

$$\begin{aligned}
\mathcal{L} = & |(\partial_\mu - ic_{c\mu})\Phi_c|^2 + V(|\Phi_c|) + \frac{1}{2K}|\partial \times c_c|^2 \\
& - i(\partial \times c_c)_\mu(4a_\mu + 2A_\mu) \\
& + \bar{\psi}_l \gamma_\mu (\partial_\mu - ia_\mu) \psi_l + |(\partial_\mu - ia_\mu - iA_\mu)\psi_b|^2 \\
& + |(\partial_\mu - ic_{p\mu})\Phi_p|^2 + V(|\Phi_p|) + \frac{1}{2K}|\partial \times c_p|^2 + i(\partial \times c_p)_\mu 2A_\mu \\
& + \frac{1}{K_{cp}}(\partial \times c_c) \cdot (\partial \times c_p). \tag{9}
\end{aligned}$$

Here  $\Phi_{c(p)}$  is a chargeon (Cooper) pair vortex field, and  $c_{c(p)\mu}$ , a chargeon (Cooper) pair vortex gauge field mediating interactions between the vortices.  $V(|\Phi_{c(p)}|)$

is a vortex potential including vortex mass and self-interaction terms.  $K = \frac{K_{sp}K_{bp}}{K_{sp}+K_{bp}}$  is a renormalized phase stiffness resulting from the coupling between the chargeon pair and Cooper pair fields, i.e.,  $\kappa_{cp}(\partial_\mu \phi_c + 4a_\mu + 2A_\mu)(\partial_\mu \phi_p - 2A_\mu)$ .  $K_{cp} = \frac{K_{sp}K_{bp}}{K_{sp}-K_{bp}}$  is a residual coupling between the two vortex gauge fields. The only remaining coupling  $\frac{1}{K_{cp}}(\partial \times c_c) \cdot (\partial \times c_p)$  between the Cooper pair sector and chargeon pair sector is irrelevant in the renormalization group sense. The scaling dimension of the coupling strength is  $\left[\frac{1}{K_{ac}}\right] = -1$  and thus the strength of the coupling diminishes as the cut-off energy gets smaller in the renormalization group procedure. As a result the Cooper pair sector and chargeon pair sector are completely decoupled.

- [10] E. Fradkin, S. H. Shenker, phys. Rev. D **19**, 3682 (1979); references therein.
- [11] O. I. Motrunich and T. Senthil, Phys. Rev. Lett. **89**, 277004 (2002); T. Senthil and O. I. Motrunich, Phys. Rev. B **66**, 205104 (2002).
- [12] A. Sudbo et al., Phys. Rev. Lett. **89**, 226403 (2002); J. Smiseth et al., Phys. Rev. B **67**, 205104 (2003).
- [13] N. Nagaosa and P. A. Lee, phys. Rev. B **61**, 9166 (2000); references therein.
- [14] D. H. Kim and P. A. Lee, Annals Phys. **272**, 130 (1999); references therein.
- [15] T. W. Appelquist and R. D. Pisarski, Phys. Rev. D **23**, 2305 (1981).
- [16] H. Kleinert et al., Phys. Rev. Lett. **88**, 232001 (2002); H. Kleinert et al., Nucl. Phys. B **666**, 361 (2003); The anomalous kinetic energy ( $\sim \sqrt{q^2}$  with  $q$ , momentum) of the gauge field  $a_\mu$  is expected to result in the  $CDT$  as a  $KT$  type in  $(2+1)D$ . The critical coupling is shown to be  $g_c = \frac{1}{12\pi^2}$ . In our case of  $N_s = 2$  and  $N_b = 1$   $g_{eff} = \frac{8}{3}$  is obtained and  $g_{eff} \gg g_c$  is satisfied. This strong coupling region corresponds to the confinement phase of the internal charge  $\bar{e}$ . In our case the deconfinement of the internal charge is expected to exist when the chargeon pair is condensed.
- [17] T. Appelquist et al., Phys. Rev. Lett. **60**, 2575 (1988).
- [18] Owing to the massless holon excitations the kinetic energy of the internal gauge field  $a_\mu$  becomes  $\frac{N_s+N_b}{16}(\partial \times a) \frac{1}{\sqrt{-\partial^2}}(\partial \times a)$  in the  $1/N$  expansion. We note that  $N_s$  in the absence of the holon field is replaced by  $N_s + N_b$  in the presence of that. In our case the flavor number of the holon quasiparticle is  $N_b = 1$ . Thus the critical flavor number of the Dirac fermion is obtained to be

$$N'_c = N_c - N_b \approx 2.24.$$

- [19] M. Hermele et al., cond-mat/0404751.
- [20] Internal gauge charge  $4\tilde{e}$  causes quartic anisotropy term for the chargeon pair vortex fields in the presence of instanton excitations. The quartic anisotropy term is irrelevant at a critical point. See T. Senthil et al., Science **303**, 1490 (2004); T. Senthil et al., cond-mat/0311326; references therein.
- [21] M. Franz and Z. Tesanovic, Phys. Rev. Lett **87**, 257003 (2001); M. Franz et al., Phys. Rev. B **66**, 054535 (2002).
- [22] Ar. Abanov, A. V. Chubukov, and J. Schmalian, cond-mat/0107421.
- [23] C. M. Varma, cond-mat/9607105; Catalin Pascu Moca, cond-mat/9911330; references therein.
- [24] S. Chakravarty et al., Phys. Rev. B **63**, 094503 (2001); C. Nayak and E. Pivovarov, cond-mat/0203580.